

Perspective and Renaissance Art, 2018 Edition, Version 1.0

Patrick G Horneker

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1 Introduction

Perspective and Renaissance Art was originally written and developed during the Fall 1988 semester for the Mathematics Colloquium class at Valparaiso University.

The project involved the resources of both the Mathematics **and** Art Departments.

1.1 There is Mathematics in Everything

There were a few people in the audience that did not think this was important¹ and not mathematics.

I beg to differ.

While computer science and mathematics were a major part of my coursework, it was the art history courses I have taken that gave me the inspiration to do this project for that semester.²

The Renaissance period to this day still is my favorite period in art history. It is this period that I used as proof that mathematics is used everywhere, even in places where we do not normally think about it.

The three artists I chose to represent that period in this document should be well known (even to those without a background in the arts).

1.2 The Future of this Document

I have kept the original documents over the years, and have scanned the original pages, including the extensive outline. At that time, I had used several typewriters to create the original document.

Initially, I am transcribing the document into Texmaker (running under PCLinuxOS³) as a \LaTeX document. As I transcribe, the text will be updated as I see appropriate.

Where this goes from here is anybody's guess. The original document can be viewed online as a PDF file.⁴

¹as indicated by one of the comments I received for feedback.

²Those courses were also my favorite courses I have taken during my time at the university.

³<http://www.pclinuxos.com>

⁴<http://horneker.com/pdf/perspective-and-renaissance-art-original-text.pdf>

2 Opening Statement

"The Renaissance artist wanted to see life from varying viewpoints and to transcend the heretofore ecclesiastic, finite way of thinking and existing whereby men were enjoyed to see in nature a sign of God's onnipresence and the truth of religion."

This statement from *The Painter's Mind* by Romare Bearden and Carl Holtz defines the concept of Renaissance Art. The term *renaissance* is defined as "rebirth". Before the time of Galileo, Copernicus and Leonardo da Vinci, the ideas and values people had in society were influenced by religion.⁵ No thought was given to any kind of reality except for the intuition that religious figures had all political power, that is, people remained outsiders to the mathematical and scientific worlds. They believed everything the government told them.⁶ The church and state were the same institution.

The art produced at the time depicted the religious figures by enlarging them out of proportion, stretching them so they look taller, illuminating them, or other techniques to invoke the feeling that these figures are superhuman and cannot be challenged. With Renaissance Art, no longer are important figures depicted out of proportion or distorted in the image. Everything is proportional according to the actual distance between you, the viewer of the art, and the object.⁷

During the Renaissance Period, people's attitudes changed accordingly. People began to wonder how the world really works. No longer did people believe that important (religious) figures had all the power.⁸

Update: *We sure could use a new Renaissance Period right now, if nothing else to get back to a time when people were thinking for themselves instead of corporate owned media doing the thinking for them.*

3 Introduction to Perspective

Since Renaissance Art's main purpose is to show reality, the concept of perspective is a major point that needs to be discussed. Perspective is defined according to the *Pictorial Cyclopedia of Photography* as a

"graphic device which artists use to reproduce the appearance of scenes and solid objects on a flat surface."

In other words, *perspective* is a two dimensional representation of a three dimensional object. This representation is found in virtual space. Virtual space is that which can only

⁵These days, the same influence comes from people's dependence on smartphones, if you consider the smartphone as a religion.

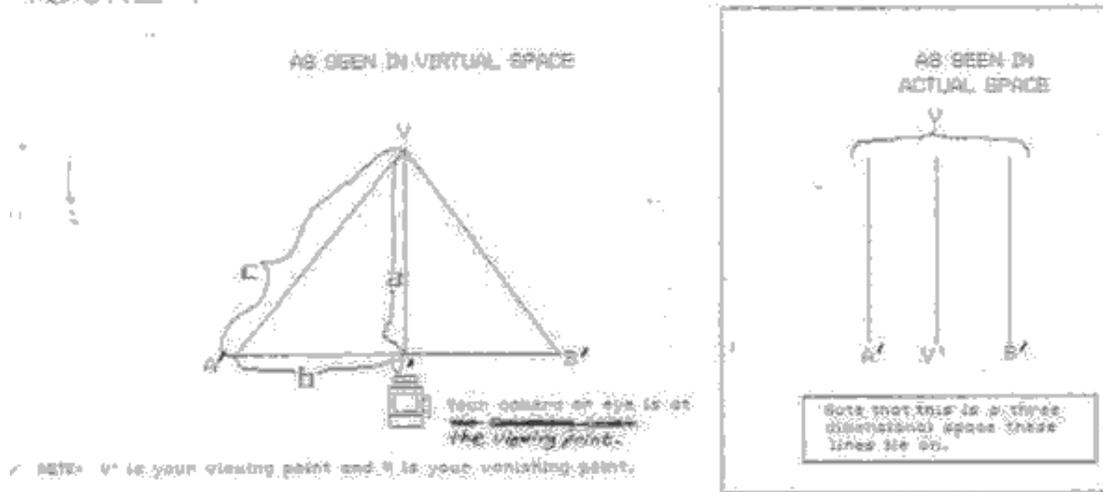
⁶One could argue this to be true for today's corporate owned news media.

⁷This is the simplified definition of the term *proportion*.

⁸The original document contained the deleted sentence, "The mathematical and scientific worlds were involved in the explanation of why things happen." I decided to delete it after reviewing the statement, and found that this was a Duh! statement. What was I thinking then?

be seen, yet not felt.⁹ In actual space¹⁰, the objects not only can be seen by the eye, but felt.¹¹

FIGURE 1



Given a plane and parallel lines on that plane, a two dimensional representation of this configuration forms an isosceles triangle. The point where the two parallel lines converge is called the *vanishing point*. Let us label this point V' , and the point where we are in virtual space should be labelled V . If we were to add additional parallel lines to the actual space, each line would meet at the same point we called V' in virtual space (the two dimensional representation). Because each of these lines in actual space is parallel, it makes sense that their lengths are equal.

Because the virtual space is a plane, the length of any of these lines is determined by the (ever so famous) *Pythagorean Theorem*, that is:

$$c = \sqrt{(a^2) + (b^2)}$$

where a is the distance between V and V' , b is the distance between where that chosen line crosses the base where we are standing and V , and c is the actual distance of the line we are measuring.

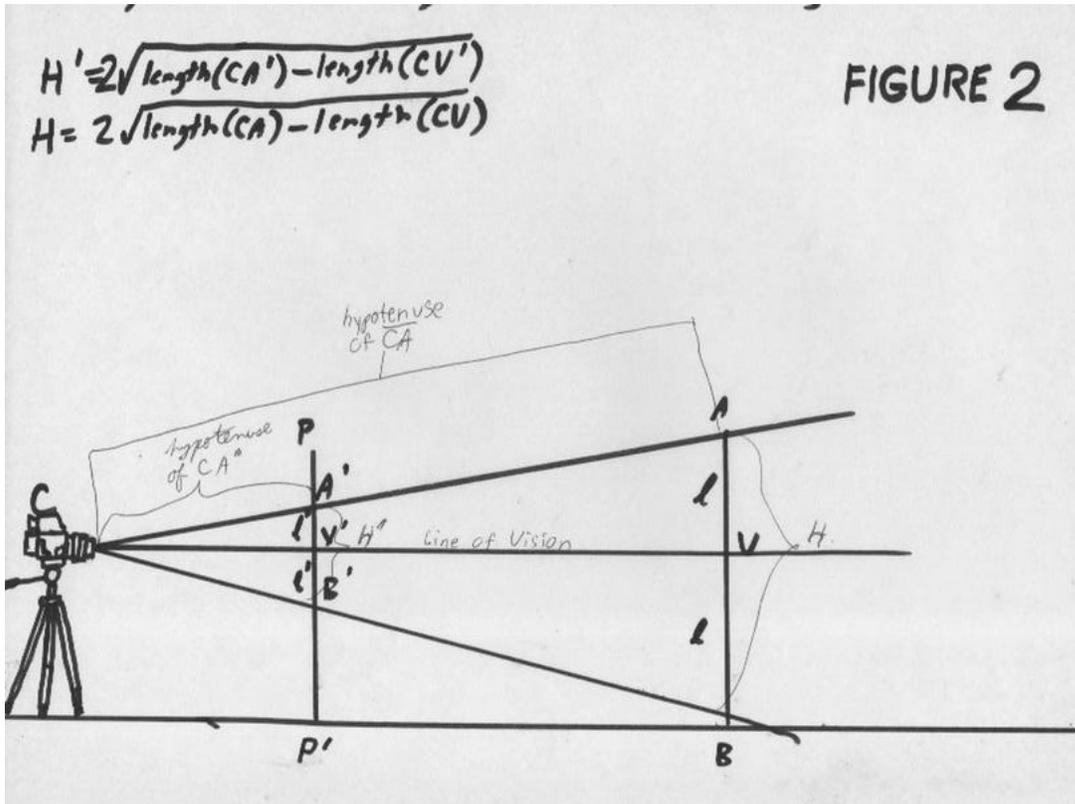
Suppose we line up three objects that are similar in size anywhere between V and V' (in the actual space). From *Codex Atlantico #176* of Leonardo da Vinci's notebooks, da Vinci states:

⁹Virtual reality, in hardware and software form, leverages this concept.

¹⁰The real world we live in,

¹¹Makes sense, right?

"The true knowledge of the form of an object becomes gradually lost in proportion as distance decreases its size."



This is what happens to the image as the object is moved away from the camera.¹² The variation of the size of the objects is the concept of scale relationships. Let us take an object represented by line **AB** (see the above figure). By da Vinci's definition (in Codus Atlantico #176), the object represented by line **AB** is shown on the image as line **A'B'**. This object appears to be smaller because we are looking at it in virtual space (i.e. on a two dimensional surface). To show this relationship, we construct two lines, one from **A** to **C** and one from **B** to **C**. Where line **AC** intersects the view of the image represented by line **PP'**, we will call **A'**. Where **BC** intersects **PP'**, we will call **B'**. Next, we construct a line *perpendicular* to both **AB** and **PP'**. At **PP'**, we will call that intersection **V'**. At **AB**, we will call that intersection **V**. This line (**VV'**) represents your line of vision. The length of **A'B'** is the apparent height of the object represented by **AB** in the image. We can now find the apparent height of the object located anywhere. Using line **AB**, we will set up a formula to find a constant. The constant represents the scale factor between the actual object and the virtual object on the image in terms of its height. Let **C** be the constant such that:

$$C = (CV' \div CV) = (CA' \div CA) = (CB' \div CB)$$

(Rather obvious, isn't it?) We then apply this constant to the ever so famous Pythagorean

¹²Likewise, the opposite happens when you move that same object towards the camera.

Theorem such that

$$CD = C\sqrt{(A^2) + (B^2)} = \sqrt{(C^2)(A^2) + (C^2)(B^2)} = \sqrt{(CA)^2 + (CB)^2}$$

which shows that even formulas like this work in proportion. Since the *apparent* height of the object is smaller than the *actual* height of the object, C will always be between 0 and 1, not including 0.

4 Leonardo Da Vinci

Leonardo da Vinci was born April 15, 1452 at the village of Vinci.¹³ In a time where feudalism¹⁴ was the political setting of Europe, education was limited only to the nobleman. Leonardo never had a formal education, but he learned to read, write, learn mathematics and Latin.¹⁵ At the age of 16, he moved to Florence, where his Renaissance artwork originated. He was an apprentice for Verocchio, a master of painting and statues. In 1483, Leonardo left Florence for Milan on an invitation sent by Ludovico Sforza, then governor of Milan. Florence left Leonardo intellectually frustrated. The Platonic Academy has a sense of superiority so great, it expressed Leonardo's failure to be recognized by his native city.¹⁶ He was appointed a member of the court, and was commissioned for small jobs, yet his main interests were in science and engineering. Between 1495 and 1498, *The Last Supper* was painted (as a wax fresco). The year the fresco was finished, Leonardo met Fra Luca Pacioli, a distinguished mathematician. Fra Luca Pacioli knew about Leonardo da Vinci's interest in mathematics and mechanics. The two collaborated on *Divina Proportione*, a book on perspective.¹⁷ Leonardo illustrated the book with his drawings. In 1499, political turmoil hit Milan and caused Leonardo da Vinci and Fra Luca Pacioli to leave Milan. Leonardo took his library which included the Bible, and books on history, Latin and mathematics. In October 1499, King Louis XII of France took over Milan. Leonardo da Vinci and Fra Luca Pacioli were well received in Venice. Three years later (in 1502), King Louis XII liked *The Last Supper* so much, he invited da Vinci to enter his service. At the same time, Cesare Borgia of Rome offered a better deal for da Vinci. He stayed in Rome for one year, and then returned to Florence, where he began painting the *Mona Lisa*.¹⁸ It was in Amboise, France where Leonardo da Vinci spent his final days. Leonardo made his Last Will and Testament on April 23, 1519 and finally passed away of natural causes on May 2, 1519 at age 67.

The Last Supper, painted between 1495 and 1498 contains elements of perspective.

5 The Last Supper

The Last Supper was a wax fresco painted by Leonardo da Vinci between 1495 and 1498. According to da Vinci, perspective is defined as:

¹³His full name was Leonardo di ser Piero da Vinci. The "da Vinci" portion indicates his birthplace.

¹⁴Which shares many of the same characteristics as imperialism.

¹⁵Which was quite impressive, since the status quo only allowed the privileged class to have an education.

¹⁶...or in Donald Trump's words, "You're Fired!"

¹⁷The term perspective here refers to the mathematical definition as explained in the previous section.

¹⁸The *Mona Lisa* was started in Florence, but the painting was finished in Amboise in France.

Perspective is the rational law by which experience confirms that all objects transmit their image to the eye in a pyramid of lines. Bodies of equal size produce angles that are more or less acute depending on their respective distances. I call "pyramid of lines" the lines that emanate from the surfaces and outlines of the bodies and, as they converge from a distance, and in a common point. We call a point that which cannot be divided in any way, and that point, situated in the eye, receives in itself the apexes of all the pyramids.

The lines of perspective in this and other Renaissance paintings with elements of perspective are determined by doing this exercise.

1. Take a sphere **S** and intersect it with a plane **P**. This plane must be intersected at the center of the sphere. Call this center point **S'**. The resulting figure on the plane is a circle with **S'** at the center.
2. Divide that circle into pie segments. You accomplish this by drawing line segments across the circle, making sure that the line segments are drawn from the edge of the circle to **S'**.
3. Reshape the circle into a rectangle while retaining all the intersections of the drawn lines. This is how perspective is shown in works of art.¹⁹

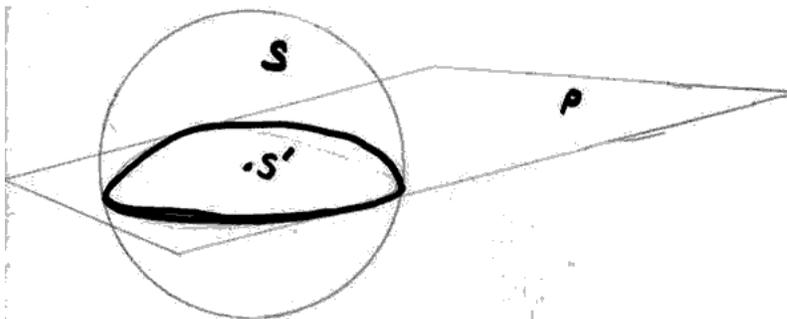
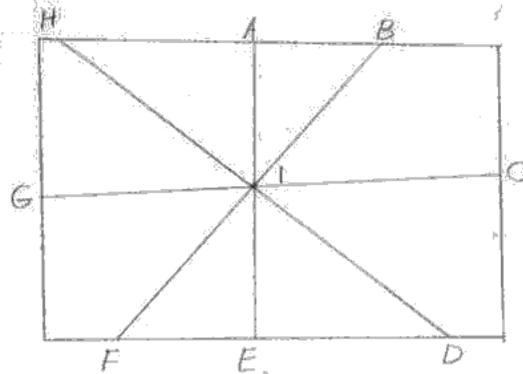
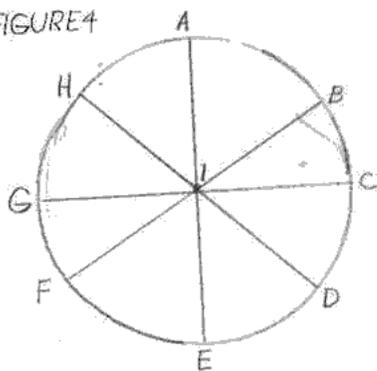


FIGURE 5

FIGURE 4



¹⁹Especially Renaissance Art.



Ultima Cena (The Last Supper) by Leonardo da Vinci - High resolution scan by <http://www.haltadefinizione.com/> in collaboration with the Italian ministry of culture. Scan details, Public Domain, <https://commons.wikimedia.org/w/index.php?curid=3032252>

In *The Last Supper*, S' lies on the left eye of Christ.²⁰ To further show Leonardo da Vinci's mathematical background, the figures (the Twelve Apostles) are symmetrically located and Christ is at the center. The key lines of perspective are shown at the corners joining the walls and the ceiling on both sides of the room, the line showing the top of the doorways, and the lines drawn between Christ and the apostles nearest to Christ. Da Vinci also positions the apostles so that everyone is looking at Christ at the same eye where S' is located. To further show perspective, notice how rapidly the size of the room shrinks as you look further into the room. The concept of scale relationships is evident in the mountains just outside the room compared to the chair in front of Christ.

When the lines of perspective are drawn through this work of art, there are many symmetric elements. When a vertical line is drawn through the painting at the designated vanishing point, the lines on the left side are a mirror image of the lines on the right side of the painting. The symmetry of this division is used by Leonardo da Vinci to show Christ is an important figure, yet as human as the rest of us. The centrality of his location shows that Christ is important in religious life. The Twelve Apostles are just as important since they dominate the foreground as well. These figures are arranged according to importance (according to the Bible). Perspective is used to emphasize importance by bringing what is important in the foreground and leaving the rest either on a side or in the background.

²⁰This is also the location where S' lies when the exercise is completed.

6 Raphael

Raffaello Sanzio da Urbino (commonly known as Raphael), was born in April 6, 1483 in the village of Urbino.²¹ His father got him interested in painting during his childhood.²² At age 17, Raphael worked with his new master Perugino. *The Marriage of the Virgin* was influenced by Perugino's version of *Christ Delivering the Keys of the Kingdom to St. Peter*. *The Marriage of the Virgin* was painted in 1504. Four years later, Pope Julius II invited Raphael to decorate the Vatican. In 1509, Raphael painted *The School of Athens*. It covered one of the walls of the Stanza della Signatura. The other three frescoes were similar in nature. *The School of Athens* represented Philosophy, *The Dispute of the Sacraments* represented Theology, *Parmassus* represented Poetry, and *Pope Gregory IX Delivering the Decretals* representing Law. For this document, only the *School of Athens* will be discussed.²³ Raphael was made chief architect in 1514 for the St. Peter's Basilica where Pope Gregory's tomb is located. His pupils helped complete the pictures he created.

On April 6, 1520, Raphael passed away at the age of 37.²⁴ According to a Wikipedia article on Raphael,²⁵

Whatever the cause, in his acute illness, which lasted fifteen days, Raphael was composed enough to confess his sins, receive the last rites, and to put his affairs in order. He dictated his will, in which he left sufficient funds for his mistress's care, entrusted to his loyal servant Baviera, and left most of his studio contents to Giulio Romano and Penni. At his request, Raphael was buried in the Pantheon.

In my opinion, that illness could well have been caused by the excessive exposure to materials he used to create his artwork.

The Marriage of the Virgin and *The School of Athens* contain elements of perspective. *The School of Athens* subjects include Euclid with Raphael listening to Euclid discussing his new theorem.

7 The Marriage of the Virgin

The Marriage of the Virgin was painted in 1504. It was influenced by Perugino's *Christ Delivering the Keys of the Kingdom to St. Peter*.

²¹Since *da Urbino* means "from Urbino", we can then say that Raphael's surname is Sanzio.

²²Both his parents passed away before Rafael reached the age of 12.

²³I may decide to include any of the others in later versions of this document.

²⁴Which was quite premature when compared to Leonardo da Vinci. Interesting thing, though is that his birthdate and death date are both on **Good Friday**.

²⁵<https://en.wikipedia.org/wiki/Raphael>



Spoznalizio (The Marriage of the Virgin) By Raphael - [1], Public Domain, <https://commons.wikimedia.org/w/index.php?curid=4800857>



Entrega de las llaves a San Pedro (Christ Delivering the Keys of the Kingdom to St. Peter) By Pietro Perugino - See below., Public Domain, <https://commons.wikimedia.org/w/index.php?curid=38902703>

Rafael simplified the background by including only the central building, namely *St. Peter's Basilica*, of which Rafael was the chief architect.²⁶ Many lines of perspective project from this vanishing point. The important characters in this painting are in the immediate foreground. One must look down from the horizon to see these figures. All figures in the foreground are about the same size as they are in the same general area in virtual space. The scale relationships are evident in the background as passers by are much smaller than the people at the wedding. They are further away from the viewer so true perspective is evident. *St. Peter's Basilica* looks larger because it *is* a large building and not to show importance.²⁷

Unlike Leonardo da Vinci's *The Last Supper*, the vanishing point (**S'**) is **not** in the exact center. When one looks at a painting, the visual center is slightly above the actual center because of the way we see. Raphael corrects this by placing **S'** above the actual center of the canvas. He forces us to look down at the actual center by placing the Virgin at the bottom of the painting.

Balance is obtained by mathematically placing the figures and the building centered on a vertical line drawn in the horizontal center of the canvas. Perguino retained balance the same way in *Christ Delivering the Keys of the Kingdom to St. Peter*²⁸, along with two other buildings in that painting. The buildings were located, one to each side of the basilica, to emphasize the central building, namely *St Peter's Basilica*. The balance in the figures happens only in the foreground.

One other concept happens in these paintings. In simple perspective, an isosceles triangle is formed in both paintings. The following exercise will show how Raphael used this triangle in *The Marriage of the Virgin*:

- Let **A** be the apex of the building.
- On the right side of the painting, find a point **B** and connect it to **A**.
- Construct a horizontal line crossing the figures. At the left edge of the painting, call the intersecting point **C**.
- Construct a line between **A** and **C**.

This is the triangle Raphael used when composing this painting. Notice that the center of this triangle is where the vanishing point would normally be. If you have constructed the triangle crossing the figures at the neck level of both the preacher and the Virgin, the center point is where Raphael located his center point on the painting. Notice how close that triangle is to being equilibrium. Mathematically, that is a perfect way to compose a picture using a triangle.²⁹

Perugino used a triangle whose angle at the apex of the building is obtuse, i.e. an angle whose measurement is more than 90 degrees (and less than 180 degrees). Raphael does not let this happen since an angle like that tends to deemphasize the importance of the Virgin. Yet, reality is retained by proportionally sizing the figures.

²⁶As Raphael's name is shown on the building.

²⁷One could argue the true importance of *St Peter's Basilica*, but that is a matter for another subject.

²⁸Hence the inclusion of *both* paintings in this document.

²⁹Which is what Raphael intended.

8 The School of Athens



Scuola di Atene (The school of Athens) By Raphael - Stitched together from vatican.va, Public Domain, <https://commons.wikimedia.org/w/index.php?curid=4406048>

The School of Athens was painted between 1509 and 1511 at Stanza della Signatura (inside the Vatican). This is a fresco representing Philosophy. Like *The Marriage of the Virgin*, the vanishing point is not in the center. Instead, the vanishing point is located *below* where you would expect to find the vanishing point. The figures of Plato and Aristotle are located in the center and the vanishing point is centered *between the figures* as Plato and Aristotle were great philosophers (and this is a fresco about *philosophy* (and mathematics)). Raphael places the mathematicians on the lower right of the painting, and places *himself*³⁰ within the set of mathematicians listening to Euclid discussing a new theorem. The lines of perspective are shown on the ceiling where the virtual image seems to extend the Stanza della Signatura, and the floor of the room where the figures are located.

Perspective is well demonstrated in *The School of Athens*. There are several lines which go to the vanishing point. Raphael forces us to look below where the center of the work is

³⁰as a self portrait

normally located. It is here where the founders of the Renaissance philosophy are depicted. The balance is obtained by evenly distributing the crowd of mathematicians and philosophers. On the left side are all the philosophers, and on the right are all the mathematicians. This is evident in the foreground, which appears to integrate with the room where the fresco was painted. Scale relationships are evident. The heads of the people in the background appear to be smaller than the heads of the people in the foreground. The arches in the background get smaller as you look down into the painting.

No other painting demonstrates the meaning of the term "*Renaissance*" than *The School of Athens*. Before and during the Renaissance period, most art depicted religious figures. This work depicts people who are involved in a new way of thinking of the world. People previously has no concept of what perspective was, let alone know about physics, mathematics, or any of the other sciences.³¹ On Plato's side, there are ancient philosophers. To show the *renaissance*, there are scientists and mathematicians from the era on Aristotle's side.

Perspective space in this painting makes the human figures "move" naturally. Before the Renaissance Period, artists knew of no way to make human figures "move" so that the vital import depends upon the beliefs of the viewer. *Vital import* is the feeling received by the viewer from what the artist intended to show in the work of art.

9 Michelangelo

Michelangelo di Lodovico Buonarroti Simoni (or simply known as Michelangelo) was born in Capresso (near Florence) on March 6, 1475. Physically, he was small and thin, yet had amazing strength and energy. He was always interested in art. His father never wanted him to become an artist.³² In 1488, Michelangelo studied with Ghirlandaio, a Florentine painter. He studied the old masters and learned to paint. Michelangelo was hot tempered and critical. He expected perfection in art.³³ At 16, he was sent to Giovanni to study sculpture. Giovanni was in charge of supervising the Medici family garden.³⁴ While living with Giovanni, Michelangelo studied anatomy, which influenced the creation of his statue *David*. Michelangelo started sculpting in Rome. *David* was created with marble that other sculptors had trouble working with. In 1505, Pope Julius II invited Michelangelo to design his tomb. The tomb was located in the same Basilica depicted in *The Marriage of the Virgin Mary*.³⁵

Michelangelo's next project was to paint the *Sistine Chapel*.³⁶ *The Last Judgement* was painted in 1536 on the altar wall. The perspective on this painting was not as obvious as it was in Leonardo da Vinci's *The Last Supper*. The position of the people in this work determine the perspective lines. In 1547, Michelangelo was appointed architect in the *expansion* of *St. Peter's Basilica*.³⁷ The dome on that building has been the inspiration for domes on

³¹Hence, the depiction of philosophers and mathematicians shows a birth of a new way of thinking.

³²His father believed that art is for peasants.

³³Wow!

³⁴Florence was under the rule of the Medici family at that time.

³⁵It took 40 years to build that very tomb.

³⁶Which took another four years to complete.

³⁷So that makes it *two* Renaissance artists that were involved in the Basilica's existence.

most public buildings.³⁸

On February 18, 1564, Michelangelo passed away. He lived for 89 years, longer than either Leonardo da Vinci or Raphael.

10 The Last Judgement



The Last Judgement By Michelangelo - See below., Public Domain,
<https://commons.wikimedia.org/w/index.php?curid=16143987>

The Last Judgement was painted in 1536 after Michelangelo painted the ceiling on the *Sistine Chapel*. It is located on the altar wall. Although this work is Baroque, there are

³⁸Like the Capital building in Washington DC for example.

elements of Renaissance art depicted. Instead of an obvious indication of perspective such as corner lines coming together, the direction the figures looked at Christ determined the lines of perspective. As with *The Marriage of the Virgin*, the vanishing point is located *above* where the vanishing point would be. Had this work been done as a three dimensional sculpture, we would be looking at the monstrosity from an aerial perspective.³⁹ Aerial perspective is defined according to *Photography: The Professional Touch* by Bob Saxton as an

Impression of depth in a landscape photograph, largely created by haze, which renders farther planes softer than nearer ones.

In this work, the softer planes are behind Christ. The figures behind Christ become smaller as you look farther into the work. What separates this from other Renaissance art is that the figures have been twisted, but not to the extent found in pre-Renaissance artwork.

In the aerial perspective form, you would be seeing it as defined by this exercise:

1. Take one cylinder **C**, intersect two planes such that one plane is *perpendicular* to the other plane. Call these planes **P** and **P'**.
2. The result is **four** quarter cylinders. Take one of those quarter cylinders and intersect another plane **Q** such that the plane intersects at a 45 degree angle. Then add a fourth plane such that the plane is *perpendicular* to **Q**. Call this plane **Q'**.
3. The resulting intersection of **Q** and **Q'** and the quarter cylinder result in the point where you would view the artwork from.

Michelangelo used this point to make you part of the audience waiting for the final judgement to take place.

Michelangelo put a rather irregular self portrait in this work. He uses this self portrait as an attempt to show how he felt at the time this was painted. He believed the artist should make up his own rules as the artist painted. The skin symbolizes Michelangelo's hatred for artists like Leonardo da Vinci who use perspective in the conventional manner.

11 Conclusion

Before the Renaissance era, most people had no knowledge of what perspective was like. Since the Church and State were considered one and the same, the Church regulated what kind of art could be produced. The masses had no concept of what reality was like except for what the governing parties told them. In paintings for the masses, religious figures had to be exaggerated to please the powers that be.⁴⁰ As a result, artists were oppressed from knowing about the concept of space relationships, or about scale relationships.⁴¹

³⁹That would have also created quite a mess on the floor given the number of statuettes that would have to be created, and it would be very difficult to get through the exhibit without bumping into any one of those statuettes, hence the term *monstrosity*.

⁴⁰This was reworded from the original text to ensure accuracy of the text, not to mention political correctness, not that it matters.

⁴¹from a mathematical point of view.

With the Renaissance era, no longer did the powers that be have control⁴² over what kind of artwork could be produced. Elements of perspective became the norm for art produced at that time. Some artists at the time⁴³ were mathematicians since they needed to know geometry to paint anything in perspective.⁴⁴ The concept of perspective made artists (and the masses) realize that there is more to this world than religion.

The masses realized that not everything is a result of spiritual powers. Paintings done using perspective show that religious figures are just as important as the individual who makes a life as a farmer, or the merchant who strives to make a decent living in the city.

However, some artists before the Renaissance era saw perspective just by looking out the window.⁴⁵ Because the amount of space in the pre-Renaissance paintings and frescoes was limited to the point that the image was almost flat, perspective could not be shown. Hence, you see small sized figures next to extremely large figures.

The masses would not have been able to understand perspective before the Renaissance period. The people were so controlled by the Church, that they actually believed everything is a result of some kind of spiritual force. With the Renaissance era, people began to question the beliefs set by the Church. Because the Church lost substantial control over the masses, the people understood reality better.

⁴²as much as they did before the Renaissance era

⁴³Not all artists during the Renaissance period were mathematicians.

⁴⁴All you needed to know then was basic geometry, and you did not need to be a mathematician to do that.

⁴⁵Rather obvious, isn't it.